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Venkatraman, A.; Schaft, A.J. van der

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Brief paper

Full-order observer design for a class of port-Hamiltonian systems[☆]A. Venkatraman^{*}, A.J. van der Schaft

Institute of Mathematics and Computing Science, University of Groningen, PO Box 407, 9700 AK, Groningen, The Netherlands

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ABSTRACT

We consider a special class of port-Hamiltonian systems for which we propose a design methodology for constructing globally exponentially stable full-order observers using a passivity based approach. The essential idea is to make the augmented system consisting of the plant and the observer dynamics to become strictly passive with respect to an invariant manifold defined on the extended state-space, on which the state estimation error is zero. We first introduce the concept of passivity of a system with respect to a manifold by defining a new input and output on the extended state-space and then perform a partial state feedback passivation which leads to the construction of the observer. We then illustrate this observer design procedure on two physical examples, the magnetic levitation system and the inverted pendulum on the cart system.

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1. Introduction

The first attempts towards nonlinear observer design were to identify necessary and sufficient conditions on a nonlinear system for converting it into a simpler form like a linear or a bilinear system up to an output injection term, for which an observer can be easily constructed (Besancon, 1999; Hammouri & Gauthier, 1989; Krener & Respondek, 1985; Levine & Marino, 1986; Xia & Gao, 1989). See also Shim, Seo, and Teel (2003) and the references in there. Another class of nonlinear systems that was studied consisted of those in which the state-dependent nonlinearities satisfied certain conditions, like being globally Lipschitz as studied by Rajamani (1998), Thau (1973) and Tsinias (1989), being a monotonic function of a linear combination of the states as in Arcak and Kokotovic (2001), Fan and Arcak (2003) or have a bounded slope (Arcak & Kokotovic, 2001). The observer design for such systems was performed by employing quadratic Lyapunov functions. Quite recently, the observer design was studied as a problem of rendering a selected manifold in the extended state-space of the plant and observer as positively invariant and globally attractive (Astolfi, Karagiannis, & Ortega, 2008; Karagiannis & Astolfi, 2008).

Shim et al. (2003) proposes a different approach to observer design by invoking passivity based concepts. The underlying idea is to make the augmented system consisting of the plant and the

observer dynamics strictly passive with respect to an invariant set in which the state estimation error is zero. In order to establish passivity, a new input and output is defined on the extended state-space and, under some assumptions on the plant and the observer, it is proved that passivation can be achieved. It has been shown that the proposed observer, on account of its passivity property, admits a redesign which makes it robust to measurement disturbances.

In this paper we consider port-Hamiltonian systems (van der Schaft, 1999, Chapter 4) and construct globally exponentially stable full-order observers for them by following a similar approach as stated in Shim et al. (2003). Our main contribution is to identify a special class of port-Hamiltonian systems that admit such a passivity based observer and give a constructive procedure for the observer design. We allow the observer gain matrices to depend on the observer states unlike in Shim et al. (2003) (where they are assumed constant) and thus also enlarge the admissible class of nonlinear systems. Interestingly, as a part of our full-order observer construction, we obtain a globally exponentially stable reduced-order observer whose formulation shares the same philosophy as the one stated in Astolfi et al. (2008) and Karagiannis and Astolfi (2008). We illustrate our observer design procedure on two well-known physical systems, the magnetic levitation system and the inverted pendulum on the cart, and perform simulations to show the efficacy of the proposed observer.

2. Passivity based observer design for port-Hamiltonian systems

2.1. Port-Hamiltonian systems

A port-Hamiltonian system model arises from network modeling of lumped-parameter physical systems. In the case of

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^{*} Corresponding author. Tel.: +31 623728640; fax: +31 503633800.

E-mail addresses: a.venkatraman@rug.nl, aneesh@math.rug.nl (A. Venkatraman), a.j.van.der.schaft@rug.nl (A.J. van der Schaft).

independent storage elements, they are represented as:

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u, \quad (1)$$

$$y = g^T(x) \frac{\partial H}{\partial x}(x), \quad (2)$$

where $x \in \mathbb{R}^n$ are the energy variables, the smooth function $H(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ represents the total energy stored and $u, y \in \mathbb{R}^m$, $m \leq n$, are the port variables. The port variables u and y are conjugated variables, in the sense that their product defines the power flows exchanged between the system and its environment. Typical examples of such pairs would be currents and voltages in electrical circuits or forces and velocities in mechanical systems. The system's interconnection structure is captured in the $n \times n$ skew-symmetric matrix $J^T(x) = -J(x)$ while $R(x) = R^T(x) \geq 0$ represents the dissipation structure. All the matrices $J(x)$, $R(x)$, $g(x)$ have entries depending smoothly on x . In some systems, the control u could also act through the interconnection structure, that is, the J matrix in (1) can be of the form $J(x, u)$, depending smoothly on x and u . A typical example of such a situation is in electrical circuits where u controls the switching action and a PWM implementation of the control action results in a continuous valued control $u \in U \subset \mathbb{R}^m$ that approximates the behavior of the switched electrical circuit by a smooth system.

The Passivity Based Control design techniques (see Ortega, van der Schaft, Mareels, and Maschke (2001) and the references in there for more details) for stabilization of port-Hamiltonian systems are based on the availability of the measurements of state variables. Further, in some situations we may not have the accurate measurement of the port-Hamiltonian output (2). This might happen for instance in mechanical systems, where the quality of the velocity measurements (which is the port-Hamiltonian output) can be very poor, and thus we might prefer to measure the position instead of the velocity. This motivates to design observers for estimating the state variables of a port-Hamiltonian system assuming that only the output (which may or may not equal the port-Hamiltonian output) is measured.

2.2. Introduction to the special class of port-Hamiltonian systems

We consider the following special class of port-Hamiltonian systems whose dynamics can be described by the model:

$$\begin{aligned} \dot{x} &= [J(x_1, u_1) - R(x_1)] \frac{\partial H}{\partial x}(x) + g(y)u_2, \\ J &= \begin{bmatrix} J_1(x_1, u_1) & T(x_1, u_1) \\ -T^T(x_1, u_1) & J_2(x_1, u_1) \end{bmatrix}, \quad R = \begin{bmatrix} R_1(x_1) & 0 \\ 0 & R_2(x_1) \end{bmatrix}, \quad (3) \\ g &= \begin{bmatrix} g_1(y) \\ g_2(y) \end{bmatrix} \end{aligned}$$

where $x = (x_1, x_2)$, with $x \in \mathbb{R}^n$ ($x_1 \in \mathbb{R}^p$, $x_2 \in \mathbb{R}^{n-p}$) is the state, $u_1 \in U \subset \mathbb{R}^m$, $u_2 \in \mathbb{R}^m$ are the inputs where U is a compact set. We assume only x_1 to be measurable, that is the measured output is $y = x_1$ (which may or may not equal the port-Hamiltonian output $y_p = g^T(x) \frac{\partial H}{\partial x}(x)$) and that the system (3) is forward complete, that is, trajectories exist for all $t \geq 0$. The matrices $J_1 \in \mathbb{R}^{p \times p}$, $J_2 \in \mathbb{R}^{(n-p) \times (n-p)}$ are skew-symmetric, $R_1 \in \mathbb{R}^{p \times p}$, $R_2 \in \mathbb{R}^{(n-p) \times (n-p)}$ are symmetric positive semi-definite and further $T \in \mathbb{R}^{p \times (n-p)}$, $g_1 \in \mathbb{R}^{p \times m}$, $g_2 \in \mathbb{R}^{(n-p) \times m}$. We assume the matrices $J_1, J_2, R_1, R_2, T, g_1, g_2$ to be smooth in their arguments. Further, the Hamiltonian $H : \mathbb{R}^p \times \mathbb{R}^{n-p} \rightarrow \mathbb{R}$ assumes the form

$$H(x_1, x_2) = x_2^T Q x_2 + K(x_1), \quad (4)$$

where $Q^T = Q > 0$ is a constant matrix and K is a smooth

nonlinear function of x_1 . We now proceed to design under certain assumptions, a globally exponentially stable full-order observer for the system (3), (4). Although the above class of systems seems rather restricted, it does encompass a good number of physical examples as illustrated later.

2.3. Problem formulation

We start by defining a passivity based observer for the system (3) in which we introduce the concept of strict passivity of a system with respect to a manifold.

Definition 1. The dynamical system represented as

$$\dot{\hat{x}} = [J(\hat{x}_1, u_1) - R(\hat{x}_1)] \frac{\partial H}{\partial \hat{x}}(\hat{x}) + g(y)u_2 + L(\hat{x}_1)v, \quad (5)$$

$$L(\hat{x}_1) = \begin{bmatrix} L_1(\hat{x}_1) \\ L_2(\hat{x}_1) \end{bmatrix},$$

where $\hat{x} = (\hat{x}_1, \hat{x}_2)$, $\hat{x}_1 \in \mathbb{R}^p$, $\hat{x}_2 \in \mathbb{R}^{n-p}$, $v \in \mathbb{R}^p$, is a passivity based observer for the system (3) if there exist smooth matrices $L_1 : \mathbb{R}^p \rightarrow \mathbb{R}^{p \times p}$, $L_2 : \mathbb{R}^p \rightarrow \mathbb{R}^{(n-p) \times p}$ and a continuous scalar function $k : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^m \rightarrow \mathbb{R}$ such that the feedback law

$$v = L_1^{-1}(\hat{x}_1)X^{-1}\{k(y, \hat{x}, u_1)y_d + v_d\}, \quad (6)$$

with $X(\in \mathbb{R}^{n \times n}) = X^T > 0$ makes the augmented system¹ composed of (3) and (5) strictly passive with respect to the manifold $\mathcal{M} = \{(x, \hat{x}) : x = \hat{x}\}$, from the new input v_d to the new output $y_d = \hat{x}_1 - x_1$.

Definition 2. The system (3), (5) with the design input (6) is strictly passive with respect to the manifold \mathcal{M} , uniformly for all $u_1 \in U$ and $u_2 \in \mathbb{R}^m$ if there exists a storage function $S(x, \hat{x}) > 0$ for every $x \neq \hat{x}$, $S(x, \hat{x}) = 0$ on \mathcal{M} and the time derivative of S along the system trajectory satisfies:

$$\begin{aligned} \frac{\partial^T S}{\partial x}(x, \hat{x})F(x_1, x_2, u_1) + \frac{\partial^T S}{\partial \hat{x}}(x, \hat{x})F(\hat{x}_1, \hat{x}_2, u_1) \\ + k(y, \hat{x}, u_1) \frac{\partial^T S}{\partial \hat{x}}(x, \hat{x})L(\hat{x}_1)\{XL_1(\hat{x}_1)\}^{-1}y_d \leq -\alpha(\|e\|), \end{aligned} \quad (7)$$

$$\left\{ \frac{\partial^T S}{\partial x}(x, \hat{x}) - \frac{\partial^T S}{\partial \hat{x}}(x, \hat{x}) \right\} g(y) \equiv 0, \quad (8)$$

$$\frac{\partial^T S}{\partial \hat{x}}(x, \hat{x})L(\hat{x}_1)L_1^{-1}(\hat{x}_1)X^{-1} = y_d^T, \quad (9)$$

where α is a positive definite function, $e = \hat{x} - x$,

$$\begin{bmatrix} f_1(x_1, x_2, u_1) \\ f_2(x_1, x_2, u_1) \end{bmatrix} = [J(x_1, u_1) - R(x_1)] \frac{\partial H}{\partial x}(x),$$

and $F = [f_1^T, f_2^T]^T$, $f_1 \in \mathbb{R}^p$, $f_2 \in \mathbb{R}^{n-p}$.

If the augmented system is strictly passive with respect to \mathcal{M} for some functions L_1, L_2 and k , then upon letting $v_d = 0$ the manifold \mathcal{M} becomes positively invariant² and globally attractive.³ The solution of the observer design problem then follows by noting that the state estimation error is zero on \mathcal{M} . Further, the asymptotic estimate of x is then given by \hat{x} .

¹ In the sequel, we shall always use the term augmented system to refer to the system composed of (3) and (5).

² The manifold \mathcal{M} is positively invariant if $(x(0), \hat{x}(0)) \in \mathcal{M} \Rightarrow (x(t), \hat{x}(t)) \in \mathcal{M}$ for all $t \geq 0$ for every initial condition $(x(0), \hat{x}(0))$.

³ The manifold \mathcal{M} is globally attractive if, for every initial condition $(x(0), \hat{x}(0))$, the distance of the augmented state vector to the manifold globally asymptotically goes to zero, i.e., $\lim_{t \rightarrow \infty} \text{dist}\{(x(t), \hat{x}(t)), \mathcal{M}\} = 0$.

2.4. Observer design

The notion of passivity is usually associated with respect to a point in the state-space rather than a manifold and accordingly, necessary and sufficient conditions for feedback passivation have been proposed. It has been established in Brynes, Isidori, and Willems (1991) that any affine control system can be rendered strictly passive by a smooth static state feedback if and only if the system has a vector relative degree $\{1, \dots, 1\}$ and is globally minimum phase. When some of the states are not measurable, additional sufficiency conditions have been proposed for partial state feedback passivation in Jiang and Hill (1998) and Shim et al. (2003). Our situation is similar to Jiang and Hill (1998) and Shim et al. (2003) as we need to achieve strict passivity of the augmented system with respect to \mathcal{M} by using a feedback law which is independent of x_2 .

We now state two key assumptions on (3), (5) and use them to prove that:

1. There exist matrices $L_1(\hat{x}_1)$ and $L_2(\hat{x}_1)$ such that the augmented system satisfies the vector relative degree and global minimum phase conditions with respect to \mathcal{M} which are needed for static state feedback passivation.
2. The augmented system satisfies an additional nonlinear growth inequality which is sufficient to make it strictly passive with respect to \mathcal{M} by a partial state feedback law $v = L_1^{-1}(\hat{x}_1)X^{-1}\{k(y, \hat{x}, u_1)y_d + v_d\}$, which is independent of x_2 .

Assumption 1. There exists a smooth globally invertible matrix $L_1(x_1) \in \mathbb{R}^{p \times p}$ and a smooth matrix $L_2(x_1) \in \mathbb{R}^{(n-p) \times p}$ such that

$$A^T(x_1, u_1) + A(x_1, u_1) > \epsilon I_{p \times p}, \quad \epsilon > 0 \quad (10)$$

holds for all x_1 , uniformly for all $u_1 \in U$, where

$$A(x_1, u_1) = \{L_2(x_1)L_1^{-1}(x_1)T(x_1, u_1) + R_2(x_1)\}.$$

Assumption 2. There exists a smooth function $\beta : \mathbb{R}^p \rightarrow \mathbb{R}^{n-p}$ such that

$$L_2(x_1)L_1^{-1}(x_1) = \frac{\partial \beta}{\partial x_1}(x_1) \quad (11)$$

holds for all $x_1 \in \mathbb{R}^p$.⁴

We next state the following theorem.

Theorem 1. Under Assumption 1,

1. The augmented system has a vector relative degree $\{1, \dots, 1\}$ with respect to the input v and the output $y_d = \hat{x}_1 - x_1$.
2. The zero dynamics of the augmented system with respect to the output y_d renders the manifold $\mathcal{P} = \{(x_1, x_2, \hat{x}_2) : \hat{x}_2 = x_2\}$ as positively invariant and globally exponentially attractive.

Proof. We compute the derivative of y_d and see that the input v appears in it pre-multiplied by the matrix L_1 . From Assumption 1, since L_1 is invertible for all x_1 , we conclude that the augmented system has a vector relative degree $\{1, \dots, 1\}$ with respect to the input v and the output y_d .

We next see that the zero dynamics of the augmented system with respect to the output y_d , defined uniformly for all $u_1 \in U$, $u_2 \in \mathbb{R}^m$, essentially consists of (3) and the equations

$$0 = T(x_1, u_1)Q\{\hat{x}_2 - x_2\} + L_1(x_1)v, \quad (12)$$

$$\begin{aligned} \dot{\hat{x}}_2 = & \{J_2(x_1, u_1) - R_2(x_1)\}Q\hat{x}_2 - T^T(x_1, u_1)\frac{\partial K}{\partial x_1}(x_1) \\ & + g_2(y)u_2 + L_2(x_1)v, \end{aligned} \quad (13)$$

where we make use of (4). We now consider the manifold \mathcal{P} and

denote its off-the-manifold coordinate as $z = \hat{x}_2 - x_2$. Computing the derivative of z along (3), (13) and using (12) yields

$$\dot{z} = \{J_2(x_1, u_1) - A(x_1, u_1)\}Qz. \quad (14)$$

We can clearly see from (14) that the manifold \mathcal{P} is positively invariant and further if we consider the Lyapunov function $V = \frac{1}{2}z^T Qz$, then Assumption 1 verifies $\dot{V} \leq -\frac{\epsilon \lambda_m^2(Q)}{\lambda_M(Q)}V$ with λ_m , λ_M denoting the minimum and maximum eigenvalue. Thus V exponentially decays to zero with convergence rate $\frac{\epsilon \lambda_m^2(Q)}{\lambda_M(Q)}$. \square

An interesting corollary that follows from Theorem 1 is

Corollary 1. Under Assumptions 1 and 2, the dynamical system

$$\begin{aligned} \dot{\eta} = & -\frac{\partial \beta}{\partial x_1}(x_1) \left\{ [J_1(x_1, u_1) - R_1(x_1)] \frac{\partial K}{\partial x_1}(x_1) + g_1(y)u_2 \right\} \\ & - T^T(x_1, u_1) \frac{\partial K}{\partial x_1}(x_1) + g_2(y)u_2 \\ & + \left\{ J_2(x_1, u_1) - R_2(x_1) - \frac{\partial \beta}{\partial x_1}(x_1)T(x_1, u_1) \right\} Q\{\eta + \beta(x_1)\} \end{aligned}$$

where $\eta \in \mathbb{R}^{n-p}$, is a reduced-order observer⁵ for x_2 and the dynamics of $\{x, \eta\}$ renders the manifold $\mathcal{N} = \{(x_1, x_2, \eta) : \eta = x_2 - \beta(x_1)\}$ positively invariant and globally exponentially attractive. The asymptotic estimate of x_2 thus is $\eta + \beta(x_1)$.

Proof. The off-the-manifold coordinate is $z = \eta - x_2 + \beta(x_1)$, which upon differentiating along the system dynamics yields

$$\dot{z} = \{J_2(x_1, u_1) - R_2(x_1) - \frac{\partial \beta}{\partial x_1}(x_1)T(x_1, u_1)\}Qz. \quad (15)$$

Using Assumptions 1, 2 and employing the Lyapunov function $V = \frac{1}{2}z^T Qz$, we can prove that (15) is globally exponentially stable. The asymptotic estimate of x_2 would then be $\eta + \beta(x_1)$. \square

Remark 1. The notion of vector relative degree is usually defined with respect to the output and the total input of the system, which for our augmented system (3)–(5) would be u_1, u_2 and v . However, our idea is to design the input v by the feedback law (6) such that the augmented system becomes strictly passive with respect to the input v_d and the output y_d , uniformly for all $u_1 \in U \subset \mathbb{R}^m$ and $u_2 \in \mathbb{R}^m$. In other words, we consider v as our design input and the other inputs u_1, u_2 can be any functions of time or state or both, belonging to their respective domains. Hence, we use the concept of vector relative degree between y_d and v , which is a small modification of the definition that is usually found in the literature (Shastri, 1999).

Remark 2. The zero dynamics of the augmented system with respect to the output y_d given by (3), (12), (13), differs slightly from the usual understanding of zero dynamics in the sense that the inputs u_1, u_2 still remain in our equations. Once again, as already stated in the previous remark, we consider v to be the design input and define the zero dynamics uniformly for all $u_1 \in U \subset \mathbb{R}^m$ and $u_2 \in \mathbb{R}^m$.

Remark 3. Assumption 1 involves finding matrices $L_1(x_1), L_2(x_1)$ such that the augmented system has a vector relative degree $\{1, \dots, 1\}$ and is globally minimum phase with respect to \mathcal{M} while Assumption 2 states that the quantity $L_2(x_1)L_1^{-1}(x_1)$ has to be integrable and satisfy (11) for some function $\beta(x_1)$. Designing such state dependent matrices that satisfy (10) and (11) would

⁴ For $p = 1$ this assumption is always satisfied.

⁵ The approach to observer design as a problem of rendering an invariant manifold in the extended state-space of the plant and observer as attractive has been detailed in Astolfi et al. (2008) – see also the references in there.

involve solving a set of algebraic and partial differential equations respectively and is usually a difficult task. Shim et al. (2003) studies the observer design problem by restricting L_1, L_2 to be constant matrices in which case Assumption 2 is trivially satisfied with $\beta(x_1) = L_2 L_1^{-1} x_1$ and hence narrows the applicable class of nonlinear systems. Indeed, as we show later in our examples, whenever T is a constant matrix, letting L_1, L_2 to be constant would suffice for the observer design, whereas in situations where T depends on x_1 , it is natural to allow L_1, L_2 to depend on x_1 in order to satisfy Assumption 1.

Remark 4. A very recent article Karagiannis and Astolfi (2008) presents a full-order observer design strategy for nonlinear systems that are affine in the unmeasurable state. The observer design is completed by introducing a dynamic scaling parameter which simplifies an integrability condition similar to our Eq. (11) in Assumption 2. However, the constructive procedure outlined in the article for computing the function β seems to involve some lengthy computations.

We next state a theorem to prove that the system (3), (5) admits a partial state feedback v (independent of x_2) that renders it strictly passive with respect to \mathcal{M} and also leads to the construction of the full-order observer.

Theorem 2. Under Assumptions 1 and 2,

1. The system (3), (5) expressed in the coordinates $(x_1, x_2, \zeta_1, \zeta_2)$ where

$$\zeta_1 = \hat{x}_1 - x_1, \quad (16)$$

$$\zeta_2 = \hat{x}_2 - x_2 - \{\beta(\hat{x}_1) - \beta(x_1)\} \quad (17)$$

assumes its global normal form⁶ with respect to the input v and the output y_d .

2. Under the additional assumption $g_1 \equiv 0$ in (3), there exist non-negative scalar functions $f_1(\zeta_1, \hat{x}_1, \hat{x}_2, u_1)$, $f_2(\zeta_1, \hat{x}_1, \hat{x}_2, u_1)$ such that the feedback law

$$v = L_1^{-1}(\hat{x}_1)X^{-1}\{-[\delta + f_1 + f_2]\zeta_1 + v_d\}, \quad (20)$$

where X was introduced in (6), $\delta(\in \mathbb{R}) > 0$, makes the system strictly passive with respect to the manifold \mathcal{M} , uniformly for all $u_1 \in U$, $u_2 \in \mathbb{R}^m$, from the input v_d to the output ζ_1 with the storage function being given by $W(\zeta) = \frac{1}{2}\zeta_2^T Q \zeta_2 + \frac{1}{2}\zeta_1^T X \zeta_1$.

Proof. We begin by defining the functions

$$F_i(\zeta_1, \zeta_2, x_1, x_2, u_1), \quad i = 1, 2, 3,$$

with $F_1 \in \mathbb{R}^p$, $F_2 \in \mathbb{R}^{n-p}$, $F_3 \in \mathbb{R}^{n-p}$ as:

$$\begin{bmatrix} F_1(\zeta_1, \zeta_2, x_1, x_2, u_1) \\ F_2(\zeta_1, \zeta_2, x_1, x_2, u_1) \end{bmatrix} = \begin{bmatrix} f_1(\hat{x}_1, \hat{x}_2, u_1) - f_1(x_1, x_2, u_1) \\ f_2(\hat{x}_1, \hat{x}_2, u_1) - f_2(x_1, x_2, u_1) \end{bmatrix}, \quad (21)$$

where the matrices f_1, f_2 were introduced in Definition 2 and

$$\begin{aligned} F_3(\zeta_1, \zeta_2, x_1, x_2, u_1) &= \frac{\partial \beta}{\partial \hat{x}_1}(\hat{x}_1)f_1(\hat{x}_1, \hat{x}_2, u_1) \\ &\quad - \frac{\partial \beta}{\partial x_1}(x_1)f_1(x_1, x_2, u_1). \end{aligned} \quad (22)$$

We next compute the dynamics of ζ_1 and ζ_2 as

$$\begin{aligned} \dot{\zeta}_1 &= F_1(\zeta_1, \zeta_2, x_1, x_2, u_1) + L_1(\hat{x}_1, u_1)v, \\ \dot{\zeta}_2 &= \{F_2 - F_3\}(\zeta_1, \zeta_2, x_1, x_2, u_1), \end{aligned} \quad (23)$$

⁶ A dynamical system with input u and output y of the same dimension, is said to be expressed in its global normal form if its dynamics can be written down in some suitable coordinates (z, y) as

$$\dot{z} = f_{11}(z) + f_{12}(z, y)y, \quad (18)$$

$$\dot{y} = f_{21}(z, y) + f_{22}(z, y)u, \quad (19)$$

where the square matrix $f_{22}(z, y)$ is invertible for every (z, y) .

where we have used (21)–(22) and the fact $g_1 \equiv 0$. We note that for each, $i = 1, 2, 3$,

$$\begin{aligned} F_i(\zeta_1, \zeta_2, x_1, x_2, u_1) &= F_i(\zeta_1, \zeta_2, x_1, x_2 + \zeta_2, u_1) \\ &\quad + F_i(0, \zeta_2, x_1, x_2, u_1), \end{aligned} \quad (24)$$

and further $F_i(\zeta_1, \zeta_2, x_1, x_2 + \zeta_2, u_1) = 0$ whenever $\zeta_1 = 0$. So, there exist continuous matrix functions $A_1(\zeta_1, x_1, x_2 + \zeta_2, u_1) \in \mathbb{R}^{p \times p}$, $A_i(\zeta_1, x_1, x_2 + \zeta_2, u_1) \in \mathbb{R}^{n-p \times p}$, $i = 2, 3$, such that

$$F_i(\zeta_1, \zeta_2, x_1, x_2 + \zeta_2, u_1) = A_i(\zeta_1, x_1, x_2 + \zeta_2, u_1)\zeta_1, \quad (25)$$

for $i = 1, 2, 3$. We can thus see from (24)–(25) that the system (3), (23) is in its global normal form with respect to input v and output $y_d (= \zeta_1)$.

Next, it is always possible to find non-negative continuous scalar functions $\psi_i(\zeta_1, x_1, x_2 + \zeta_2, u_1)$, $i = 1, 2, 3$ such that,

$$\|A_i(\zeta_1, x_1, x_2 + \zeta_2, u_1)\| \leq \psi_i(\zeta_1, x_1, x_2 + \zeta_2, u_1), \quad (26)$$

holds for all $\zeta_1, x_1, x_2 + \zeta_2, u_1$, where $\|\cdot\|$ is the induced norm of any general matrix. From the form of J, R in (3) and using (4), (21) we obtain the inequality,

$$\|F_1(0, \zeta_2, x_1, x_2, u_1)\| \leq \|T(x_1, u_1)\| \|Q\zeta_2\|, \quad (27)$$

where we make use of the standard matrix norm property. We further obtain, $\|Q\zeta_2\| \leq \alpha\sqrt{\epsilon}\|\zeta_2\|$ where $\alpha = \lambda_M(Q)/\sqrt{\epsilon}$ and ϵ as introduced in (10). We then obtain the inequalities

$$\begin{aligned} \|\zeta_2^T Q \{F_2 - F_3\}(\zeta_1, \zeta_2, x_1, x_2 + \zeta_2, u_1)\| \\ \leq \alpha\{\psi_2 + \psi_3\}(\zeta_1, x_1, x_2 + \zeta_2, u_1)\sqrt{\epsilon}\|\zeta_2\|\|\zeta_1\|, \end{aligned} \quad (28)$$

$$\begin{aligned} \|\zeta_1^T X L_1^{-1}(x_1 + \zeta_1)F_1(\zeta_1, \zeta_2, x_1, x_2 + \zeta_2, u_1)\| \\ \leq \psi_1(\zeta_1, x_1, x_2 + \zeta_2, u_1)\lambda_M(X)\lambda_M(L_1^{-1})\|\zeta_1\|^2, \end{aligned} \quad (29)$$

$$\begin{aligned} \|\zeta_1^T X L_1^{-1}(x_1 + \zeta_1)F_1(0, \zeta_2, x_1, x_2, u_1)\| \\ \leq \alpha\|T(x_1, u_1)\|\lambda_M(X)\lambda_M(L_1^{-1})\sqrt{\epsilon}\|\zeta_2\|\|\zeta_1\|. \end{aligned} \quad (30)$$

We now consider the observer feedback law (20) with $f_1 = \psi_1\lambda_M(X)\lambda_M(L_1^{-1})$ and $f_2 = \alpha\{\psi_2 + \psi_3 + \|T\|\lambda_M(X)\lambda_M(L_1^{-1})\}$. We differentiate the storage function $W(\zeta_1, \zeta_2) = \frac{1}{2}\zeta_2^T Q \zeta_2 + \frac{1}{2}\zeta_1^T X \zeta_1$ along (3), (23) and use (28), (29), (30) to finally obtain

$$\dot{W} \leq -\delta\|\zeta_1\|^2 + \zeta_1^T v_d - \frac{3}{4}\epsilon\|\zeta_2\|^2 - \left\{\frac{1}{2}\sqrt{\epsilon}\|\zeta_2\| - \|\zeta_1\|f_2\right\}^2.$$

Thus, the system is strictly passive with respect to the manifold \mathcal{M} , from input v_d to the output $y_d (= \zeta_1)$ with the storage function being $W(\zeta_1, \zeta_2)$. Further, upon letting $v_d = 0$ and performing some simple computations, we get that $\dot{W} \leq -\frac{1}{c}W$ where $c = \max(\frac{\lambda_M(X)}{2\delta}, \frac{2\lambda_M(Q)}{3\epsilon})$ and hence the Lyapunov function $W(\zeta_1, \zeta_2)$ exponentially decays to zero with convergence rate $\frac{1}{c}$. \square

Remark 5. If we let $e = (e_1, e_2)$ denote the state estimation error (as also introduced in Definition 2), then the storage function W when expressed in the coordinates (x, e) takes the form $W(x, e) = \frac{1}{2}e_1^T X e_1 + \frac{1}{2}\{e_2 - \beta(x_1 + e_1) + \beta(x_1)\}^T Q \{e_2 - \beta(x_1 + e_1) + \beta(x_1)\}$. We thus obtain a Lyapunov function that depends both on the state and error coordinates, different from the quadratic error Lyapunov functions.

Remark 6. The inequalities (28)–(30) are a nonlinear growth condition on W which require the growth rate to be linearly bounded in ζ_2 . If $R_2(x_1) > 0$ then Assumption 1 holds with $L_2 = 0$. In this situation if we allow the matrices J_1, R_1 to also depend on x_2 , then by performing some simple computations we can show that W can be linearly bounded in ζ_2 provided the quantity

$J_1(x_1, x_2, u_1) - R_1(x_1, x_2)$ is globally Lipschitz⁷ in x_2 , uniformly for all $u_1 \in U$.

Remark 7. The assumption $g_1 \equiv 0$ ensures that (8) is satisfied, that is, the input u_2 is decoupled from the dynamics of (ζ_1, ζ_2) and hence the observer design is independent of u_2 . This would be the case in mechanical systems where the input is the external force applied and it appears in the dynamics of the (unmeasured) generalized momenta. When β is a linear function of its argument (as considered in Shim et al. (2003)), the assumption $g_1 \equiv 0$ can be relaxed.

Remark 8. If u_1 is a continuous time varying external signal taking values in a compact set $U \in \mathbb{R}^m$ and further has a bounded derivative, then the matrices L_1 and L_2 used in the observer dynamics can be allowed to depend smoothly on u_1 . This is also natural from the view point of having to satisfy Assumption 1 because the matrix T depends on u_1 . Shim et al. (2003) considers all plant inputs to be external time varying signals in their observer design.

In the next section, we illustrate our proposed observer design by considering some physical examples which come under the class of (3).

3. Physical examples

3.1. Magnetic levitation system

We consider the magnetic levitation system as described in Astolfi et al. (2008), consisting of an iron ball in a vertical magnetic field created by a single electromagnet, described by the port-Hamiltonian model

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} -R_2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{pmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \\ \frac{\partial H}{\partial x_3} \end{pmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \quad (31)$$

where x_1 corresponds to the flux and x_2, x_3 are the position and momentum of the ball respectively. The system's energy is given as $H(x_1, x_2, x_3) = \frac{1}{2m}x_3^2 + mgx_2 + \frac{1}{2k}x_1^2\{1 - x_2\}$ with m being the mass of the ball and k is some positive constant that depends on the number of coil turns. In (31), the ball position x_2 is scaled such that when $x_2 = 1$, the ball touches the electromagnet. To avoid this singularity, we assume that $x_2(t) < 1$, that is, the ball remains strictly below the magnet.

We assume the flux and position to be measurable while the momentum cannot be measured. Thus, (31) fits in the framework of (3). We let $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ be the state estimates and define their dynamics as in (5). If $(e_1, e_2, e_3) = (\hat{x}_1, \hat{x}_2, \hat{x}_3) - (x_1, x_2, x_3)$ denotes the error, then upon choosing $L_1 = \begin{bmatrix} 1 & 0 \\ 0 & m \end{bmatrix}$, $L_2 = [0 \ m]$ in our observer construction, we obtain the zero dynamics of (x, \hat{x}) with respect to the outputs (e_1, e_2) as $\dot{e}_3 = -\frac{e_3}{m}$. Then, computing the time derivative of the Lyapunov function $V(e_3) = \frac{1}{2m}e_3^2$ along the zero dynamics yields $\dot{V} = -\{\frac{e_3}{m}\}^2$ and hence $\epsilon = 1/m^2$. We introduce the change of coordinates $\zeta = (\zeta_1, \zeta_2, \zeta_3) = (e_1, e_2, e_3 - e_2)$ to obtain the dynamics in the global normal form. We choose the total storage function as $W(\zeta) = \frac{1}{2}\zeta_1^2 + \frac{1}{2m}\zeta_2^2 + \frac{1}{2m}\zeta_3^2$ and compute the inequalities (28), (29), (30) to get $f_1 = \frac{1}{m^2} + \frac{R_2}{k}\{|1 - \hat{x}_2| + |x_1|\}$ and $f_2 = \frac{1}{2k}|x_1 + \hat{x}_1|$. We accordingly

choose the observer feedback v as

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = - \left[\delta + \frac{1}{m^2} + \frac{R_2}{k} \{|1 - \hat{x}_2| + |x_1|\} + \frac{1}{4k^2}|x_1 + \hat{x}_1|^2 \right] \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} + \begin{pmatrix} v_{d1} \\ v_{d2} \end{pmatrix}, \quad (32)$$

and verify that $\dot{W} < e_1 v_{d1} + e_2 v_{d2}$ and hence the system is strictly passive with respect to the input (v_{d1}, v_{d2}) and the output (e_1, e_2) .

3.2. Inverted pendulum on cart system

We consider the inverted pendulum on a cart example (Acosta, Ortega, Astolfi, & Mahindrakar, 2005; Teel, 1996; Venkatraman, Ortega, Sarras, & van der Schaft, 2008) which is a two degree of freedom mechanical system. It can be modeled in the port-Hamiltonian form

$$\begin{pmatrix} \dot{q} \\ \dot{\bar{p}} \end{pmatrix} = \begin{bmatrix} 0 & T(q) \\ -T^\top(q) & 0 \end{bmatrix} \begin{pmatrix} \frac{\partial V}{\partial q} \\ \frac{\partial q}{\partial \bar{p}} \end{pmatrix} + \begin{bmatrix} 0 \\ T^\top(q)G(q) \end{bmatrix} u, \quad (33)$$

where $q = (q_1, q_2)$ with q_1 being the angle made by the pendulum with the vertical axis, q_2 being the horizontal position of the cart and $\bar{p} = (\bar{p}_1, \bar{p}_2)$ are the pseudo momenta. We obtain \bar{p} by the change of coordinates $\bar{p} = T^\top p$, where $p = M(q)\dot{q}$ are the actual momenta with $M(q)$ being the inertia matrix and $TT^\top = M^{-1}$. Refer to the paper by Venkatraman et al. (2008) for more details. The matrices $T(q)$, $G(q)$ and the potential energy function $V(q)$ are given as

$$T(q) = \begin{bmatrix} \frac{\sqrt{m_3}}{\sqrt{m_3 - b^2 \cos^2 q_1}} & 0 \\ \frac{-b \cos q_1}{\sqrt{m_3} \sqrt{m_3 - b^2 \cos^2 q_1}} & \frac{1}{\sqrt{m_3}} \end{bmatrix}, \quad G(q) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (34)$$

and $V(q) = a \cos(q_1)$. We assume that only q is measurable and see that (33) fits in the framework of (3). We next compute that $\{P(q)T(q)\}^\top + \{P(q)T(q)\} > \epsilon I$, $\epsilon = 2 \min\{1, \frac{1}{\sqrt{m_3}}\}$ where $P(q) = \nabla \beta(q)$ given as

$$P(q) = \begin{bmatrix} 1 & 0 \\ \frac{b \cos q_1}{m_3} & 1 \end{bmatrix}, \quad \beta(q) = \left[\frac{b \sin q_1}{m_3} + q_2 \right]. \quad (35)$$

Since, $P(q) = L_2(q)L_1^{-1}(q)$, we choose $L_2 = P$, $L_1 = L_{2 \times 2}$. For constructing the full-order observer we introduce the coordinates $\zeta_1 = \hat{q} - q$, $\zeta_2 = \hat{\bar{p}} - \bar{p} - \{\beta(\hat{q}) - \beta(q)\}$, where $\bar{p} = T^\top(q)p$ and $[\hat{q}, \hat{\bar{p}}]^\top$ is the estimate of $[q, \bar{p}]^\top$. Next, using standard results of functional analysis we get:

$$\|\beta(\hat{q}) - \beta(q)\| \leq \left\{ \sup_q \|\nabla \beta(q)\| \right\} \|\zeta_1\|,$$

$$\left\| T^\top(\hat{q}) \frac{\partial V}{\partial \hat{q}} - T^\top(q) \frac{\partial V}{\partial q} \right\| \leq \left\{ \sup_q \left\| \nabla \left\{ T^\top(q) \frac{\partial V}{\partial q} \right\} \right\| \right\} \|\zeta_1\|,$$

$$\|T(\hat{q}) - T(q)\| \leq \left\{ \sup_{q_1} \left\| \frac{\partial T}{\partial q_1} \right\| \right\} \|\zeta_1\|,$$

$$\|P(\hat{q})T(\hat{q}) - P(q)T(q)\| \leq \left\{ \sup_{q_1} \left\| \frac{\partial}{\partial q_1}(PT) \right\| \right\} \|\zeta_1\|.$$

If λ denotes the eigenvalue, then we obtain the following bounds:

$$|\lambda(P^\top P)| \leq 1 + \frac{b^2}{2m_3^2} + \frac{b}{2m_3} \sqrt{4 + \frac{b^2}{m_3^2}} = M_1,$$

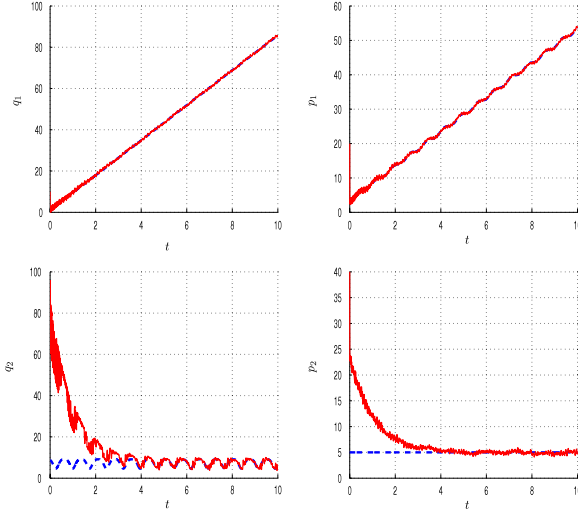
$$\left| \lambda \left(\nabla \left\{ T^\top \frac{\partial V}{\partial q} \right\} \right) \right| \leq a\sqrt{m_3} \sqrt{m_3 - b^2} = M_2,$$

⁷ A function $f(x_1, x_2, u)$ is said to be globally Lipschitz in x_2 , uniformly for all $u_1 \in U$ if there exists a non-negative scalar function $\psi(x_1, u)$ such that $\|f_1(x_1, x_2 + w_2, u) - f_1(x_1, x_2, u)\| \leq \psi(x_1, u)\|w_2\|$.

Table 1

Simulation parameters for the inverted pendulum example.

$q_1(0) = 1, \hat{q}_1(0) = 10$	$p_1(0) = 8.8513, \hat{p}_1(0) = 67.23$
$q_2(0) = 3, \hat{q}_2(0) = 20$	$p_2(0) = 5, \hat{p}_2(0) = 40$
$g = 10$	$m_3 = 1$
$b = 0.5, a = 2$	$\epsilon = 5, \delta = 10$

**Fig. 1.** Open-Loop trajectories for the Inverted Pendulum on Cart and the Observer with $u = 0$.

$$\left| \lambda \left(\frac{\partial^T T}{\partial q_1} \frac{\partial T}{\partial q_1} \right) \right| \leq \frac{m_3 b^2 (1 + b^2)}{(m_3 - b^2)^3} = M_3,$$

$$\left| \lambda \left(\frac{\partial}{\partial q_1} \{PT\} \right) \right| \leq \frac{\sqrt{m_3} b^2}{(m_3 - b^2)^{\frac{3}{2}}} = M_4,$$

$$|\lambda(T^T T)| \leq \frac{1 + m_3 + \sqrt{(1 - m_3)^2 + 4b^2}}{2(m_3 - b^2)} = M_5,$$

$$|\lambda(PT)| \leq \max \left\{ \frac{\sqrt{m_3}}{\sqrt{m_3 - b^2 \cos^2 q_1}}, \frac{1}{\sqrt{m_3}} \right\} = M_6.$$

We then use the storage function $W(\zeta_1, \zeta_2) = \frac{1}{2} \{\zeta_1^T \zeta_1 + \zeta_2^T \zeta_2\}$ and compute the inequalities (28), (29), (30) to get $f_1 = \sqrt{M_1 M_5} + \|\hat{p}\| \sqrt{M_3}$, $f_2 = \frac{1}{\sqrt{\epsilon}} \{\sqrt{M_5} + M_2 + \|\hat{p}\| M_4 + M_6 \sqrt{M_1}\}$. We accordingly design the observer feedback law given by (20) to complete the problem.

We now assume $u = 0$ (unforced system) and perform some simulations for the inverted pendulum on the cart example. The simulation parameters are shown in Table 1. We also introduce additional disturbances in the measurements of q whose maximum amplitude is equal to 1% of the maximum magnitude of the measured signals during the simulation time. We present the plots showing the system and the observer trajectories with the dashed line representing the plant state and the solid line representing the observer state. We can see that the observer is robust to the measurement disturbances and convergence is achieved (Fig. 1).

Remark 9. We refer to the paper by Venkatraman et al. (2008) for two other physical examples, namely a 3-link underactuated planar manipulator and a planar redundant manipulator with one elastic degree of freedom, both of which can be rendered linear in the unmeasured coordinates and hence follow the same observer design methodology as the inverted pendulum on cart example. The matrices L_1 and L_2 once again depend on the generalized position coordinate q .

4. Conclusion

We have proposed a passivity based full-order observer design framework for a class of port-Hamiltonian systems which leads to the construction of a globally exponentially stable observer. The idea is to render the augmented system (composed of the plant and the observer dynamics) strictly passive with respect to an invariant manifold defined on the extended state-space on which the state estimation error is zero. We also obtained as a part of the full-order observer construction, a globally exponentially stable reduced-order observer.

The observer construction is done in two steps:

- (1) Compute the observer gain matrices $L_1(\hat{x}_1)$ and $L_2(\hat{x}_1)$ by solving a set of algebraic and partial differential equations such that the augmented system has a vector relative degree $\{1, \dots, 1\}$ and is globally minimum phase with respect to the manifold \mathcal{M} .
- (2) Compute the partial state feedback law, $v(y, \hat{x}, u_1)$ by following the procedure given in the proof of Theorem 2, in order to render the augmented system strictly passive with respect to the manifold \mathcal{M} .

We finally demonstrated the observer design on two well-known physical examples, the magnetic levitation system and the inverted pendulum on cart.

Under some additional assumptions, we proved the separation principle for the proposed full-order observer when employed in closed-loop with a passivity based control (PBC) state feedback law, by using concepts from nonlinear cascaded systems theory. Refer to the internal report by Venkatraman and van der Schaft (2009) for more details.

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A. Venkatraman was born in Tiruchirappalli, in the state of Tamil Nadu, India in 1982. He obtained his Bachelor of Technology and Master of Technology degrees in Aerospace Engineering with specialization in Dynamics and Controls, from the Indian Institute of Technology Bombay, India in 2005. Since May 2006, he is a Ph.D. student at the Institute of Mathematics and Computing Science, University of Groningen in Netherlands.

He was a guest researcher at the Laboratoire des Signaux et Systèmes, Supélec, France (Sep–Oct 2007, Feb–Mar 2008) and at the Department of Electrical and

Electronic Engineering, Imperial College of London (Jan 22nd–Feb 7th 2009). His research interests are mainly in the fields of nonlinear control with focus on mechanical and electromechanical systems, robotics, optimal control with emphasis on path planning and differential geometric control theory.



A.J. van der Schaft received the undergraduate and Ph.D. degrees in Mathematics from the University of Groningen, The Netherlands. In 1982 he joined the University of Twente, where he was appointed as a full professor in 2000. In 2005 he returned to Groningen as a full professor in Mathematics.

He has served as Associate Editor for Systems & Control Letters, Journal of Nonlinear Science, SIAM Journal on Control, and the IEEE Transactions on Automatic Control. Currently he is Associate Editor for Systems and Control Letters, Journal of Geometric Mechanics, and Editor-at-Large for the European Journal of Control. He is (co-)author of a number of books, including Nonlinear Dynamical Control Systems (1990, with H. Nijmeijer), L₂-Gain and Passivity Techniques in Nonlinear Control (1996, 2000), and An Introduction to Hybrid Dynamical Systems (2000, with J.M. Schumacher). Arjan van der Schaft was an Invited Speaker at the 2006 International Congress of Mathematicians in Madrid, Spain.